BE 3600 BIOMEDICAL INSTRUMENTATION (LAB)

Experiment 10:

Signal Processing

ECG Filtering using a Digital Signal Processing Algorithm

The Continuous Fourier Transform translates analog signal information in the time domain to analog signal information in the frequency domain. The Discrete Fourier Transform does the same thing, but is applied to discrete (digital, or sampled) signals.

The signal information that is (usually) most easily determined in the time domain are the maximum and minimum amplitudes. Frequency and phase shifts may also be determined, but may prove extremely difficult in the time domain if multiple frequencies are present. For this reason, the Fourier Transform is applied. The Fourier Transform of a signal allows one to easily identify the frequencies present in a signal.

If there is a signal, $x(t) = A\cos(2\pi f_0 t + \theta_c)$ (where *A* is the amplitude, *f* is the frequency, in Hz, and θ_c is a phase shift constant, in radians, of the signal), this signal can be digitized by sampling it at a frequency of $f_s \ge 2f_0$. The digital frequency of the digitized signal x[n] is

$$F_0 = \frac{f_0}{f_s}$$

[Equation 1]

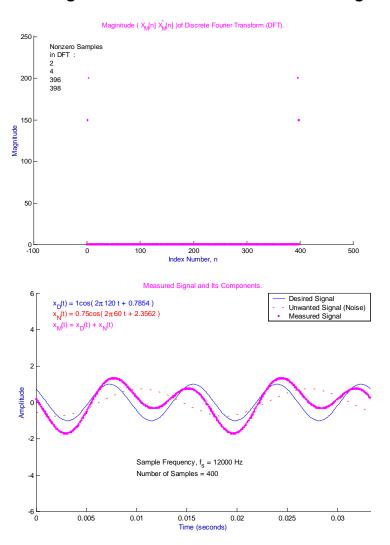
(Digitized Frequency of a sampled signal.)

A simple, analog cosine signal, x(t), can then be written as a digitized cosine signal, $x[n]=x(n\Delta T)=x(n/f_s)$, as shown below:

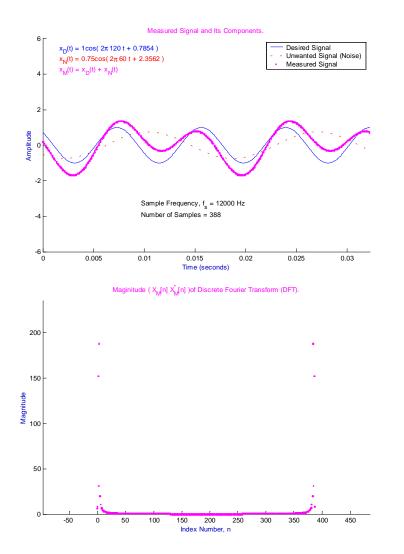
 $\begin{aligned} x(t) &= A\cos(2\pi f_0 t + \theta_c) \implies x[n] = A\cos(2\pi F_0 n + \theta_c), \ n=1,2,3,... \\ & [2.a] \qquad [2.b] \end{aligned}$ Sample at a frequency f_s Please note that in Equations 2a and 2d, a for Analog and d for Digital.) $A &= \text{Maximum amplitude} \qquad A = \text{Maximum amplitude (unchanged)} \\ f_0 &= \text{Analog frequency, in Hz} \\ F_0 &= f_0 f_s = \text{Digital frequency, in samples per second} \\ t &= \text{Time, in seconds} \qquad n = \text{An integer sample number} \\ (\dots, -2, -1, 0, 1, 2, \dots) \\ \theta_c &= \text{Phase shift constant, in radians} \\ \theta_c &= \text{Phase shift constant, in radians (unchanged)} \end{aligned}$

Discrete Fourier Transforms (DFT's) are used to analyze the frequency content of discrete signals because writing routines to compute the DFT are fairly simple. Computing a Fourier Transform or DFT by hand can be a lot of work, unless the signal in question is an impulse (a.k.a delta function), $\delta(t)$, in which case the Fourier Transform has a value of one for all frequencies. The same is of course true for the DFT of $\delta[n]$, the discretized (sampled) version of the delta function.

A minimal amount of the analog signal is required when computing its Fourier Transform in order to avoid what is called leakage or spillage. The same is true when computing the DFT of a discrete signal. To see an example of leakage (or spillage), refer to the plots on the following page. A simple way to avoid leakage is to find the period of the signal, and use an integer number of periods when computing the Fourier Transform or DFT. Example 1 : Measured Signal and Its Components, Sampled for an Integer Number of Periods, And the Magnitude of the DFT of the Measured Signal –



Example 2 : Measured Signal and Its Components, NOT Sampled for an Integer Number of Periods, And the Magnitude of the DFT of the Measured Signal (Shows Leakage, or Spillage) –



If a signal, such as an ECG, contains an unwanted frequency, or a noise signal, of a known frequency or frequencies, the noise signal can be removed by the following process :

- 1. Sample the signal to be "cleaned up" for an integer number of periods.
- 2. Compute the DFT of the signal.
- 3. Compute the indices in the DFT that correspond to the frequency or frequencies to be removed from the signal. Each frequency to be removed will have two indices associated with it in the DFT.
- 4. Set the indices corresponding to each frequency to be removed equal to zero.
- 5. Compute the real part of the Inverse Discrete Fourier Transform of the modified DFT.

Formulae for the Discrete Fourier Transform (DFT), Inverse Discrete Fourier Transform (IDFT), and computation of indices corresponding to a particular frequency, are shown below:

DFT Formula :
$$X[k] = \sum_{n=0}^{n=N-1} x[n] e^{-j2\pi \frac{nk}{N}}$$
 $k = 0, 1, 2, 3, ..., N-1.$ [Equation 3]

x[*n*] represents samples in a digitized signal containing *N* samples in time domain

X[k] represents samples in a DFT containing N samples.

IDFT Formula :
$$x[n] = \frac{1}{N} \sum_{k=0}^{k=N-1} X[k] e^{j2\pi \frac{nk}{N}}$$
 $n = 0, 1, 2, 3, N-1.$ [Equation 4]

Formula for index locations in DFT of samples corresponding to a particular frequency, f_0 :

$$k_{1f} = \frac{f_0 N}{f_s}$$
 [Equations 5a and 5b]
$$k_{2f} = N - k_{1f}$$

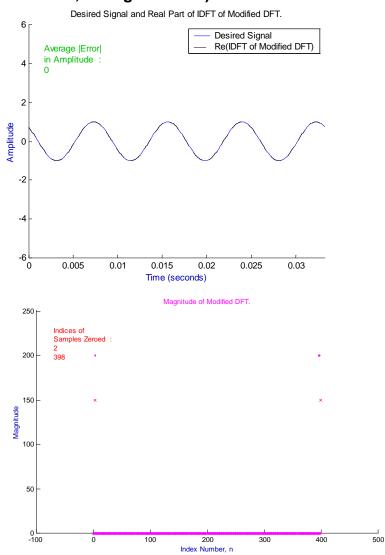
where f_0 = frequency of interest, N = number of samples, and f_s = sample frequency.

If k_{1f} and k_{2f} are not integer numbers, then they must be rounded to the nearest integer number, but if trying to remove the frequency that corresponds to k_{1f} and k_{2f} by zeroing the samples at those indices in the DFT, success is impossible. In order to be successful, k_{1f} and k_{2f} <u>must</u> be integers.

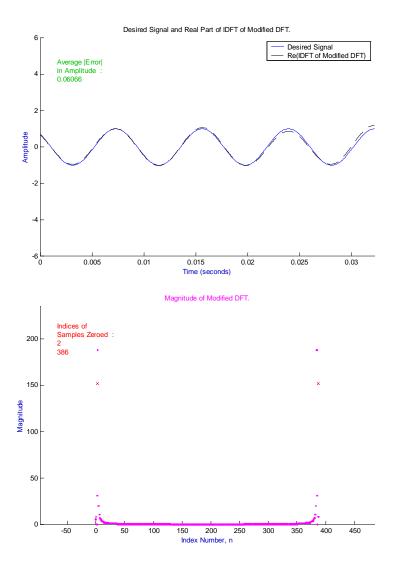
There are always two samples in the DFT that correspond to one particular frequency.

The plots of a measured signal and its components on Examples 1 and 2 above show the desired signal and the noise signal as well. The plots on Example 3 below show the magnitudes of the modified DFT of each measured signal, and the real part of the Inverse Discrete Fourier Transform (IDFT) of each modified DFT.

Example 3, Magnitude of Modified DFT of Measured Signal from Example 1, And the Real Part of the IDFT of the Modified DFT. (Red 'x' represents a sample that has been zeroed out, or removed, in original DFT.) –



Example 4, Magnitude of Modified DFT of Measured Signal from Example 2, And the Real Part of the IDFT of the Modified DFT –



Leakage, or spillage, occurs when the measured signal does not contain an integer multiple of the overall period of the signal. The overall period is the period that results from all the frequencies present in the signal. If the overall period is not known, the best way to minimize leakage or spillage is to use a very large number of samples when computing the DFT.

Directions for a computer lab exercise are as follows:

BE 3600 and 3400 ECG Filtering Exercise Using Matlab.

If you do not have access to a PC with Matlab software on it, make your T.A. aware of this.

Get a copy of the file "BE3600ECG.m" or "BE3400ECG.m" from your T.A. Have a 3.5 floppy disk or a zip disk so he/she can copy it to the disk.

The file is a Matlab script file, and you will need to put it in the Matlab "work" directory on the PC on which you will do the exercise. To do this, follow the steps below.

- 1. Log onto a PC.
- 2. Open "My Computer."
- 3. Open the "C:" drive.
- 4. Find "MatlabSR11." You may have to open "Programs" first, and it is also possible that the directory/folder you will need to open will have a different name than "MatlabSR11." If you find step five impossible, try the alternative method, described below, of getting the script file into the "work" directory/folder.
- 5. After you open "MatlabSR11," or whatever directory/folder you figure out you need, find the "work" directory/folder.
- 6. Keeping the "work" directory/folder open, make sure you have your disk with the file on it in the appropriate drive, then open that drive, be it the 3 ½ floppy drive or the zip drive.
- 7. Find "BE3600ECG.m" or "BE3400ECG.m," and right click on it. Left click on "Copy."
- 8. Now go back into the "work" directory/folder, and left click on "Edit," at the top, towards the left of the window. Then left click on "Paste." The Matlab script file should now be in the Matlab "work" directory/folder.

If you have trouble finding the directories/folders you need, try the following:

- 1. Open or run Matlab.
- 2. In the Matlab command window, left click "File," then "Open."
- 3. Matlab should be in the "work" directory already. You can try copying and pasting the file from your disk to here, or have Matlab open the file from the disk, then save it again in the "work" directory/folder.

If you still cannot get the script file into the "work" directory, get help.

Once the script file is in the "work" directory/folder, open, or run, Matlab. In the Matlab command window, type "BE3600ECG" or BE3400ECG". Do not include the quotation marks, and do not put the ".m" at the end.

Work through the exercise. Be sure to enter at least your first initial and your entire last name. Experiment with all the up and down arrows, when they first appear, next to the index numbers, to see what they do. Then use them to filter the ECG as well as possible by zeroing out the sample at two index locations. It is suggested that you calculate the two indices first. All the information necessary to do so, except the equations, will be shown on the display. When you are done filtering one signal, continue on to the next.

When you want to copy or print the information on a display, click "Pause Graphics Interface to Copy or Print Data." A new figure will be opened with a gray background, and all plots and text in black and white. If you want to change these colors, refer to the instructions in the appendix. You can copy a figure from Matlab by left clicking "File" in the figure window, then left click "Copy Figure." You can then paste it to an MS Word document, or some other document. You can print the figure on its own page by left clicking "File" in the figure window, then "Print." Select appropriate printer.

<u>REPORT</u>

- A plot of the first contaminated ECG signal, the ideal signal, the DFT of the contaminated signal, and the filtered signal, after you have zeroed the appropriate samples. When you pause the graphics interface to copy or print the data, new plots will be created automatically that will contain all the necessary information for the ECG signal on which you are currently working. To copy or print the data, left click "File" in the figure window, then left click on "Copy Figure" or "Print" to copy figure or print figure. If you copy the figure, you can paste it into an MS Word document. Do this before you copy anything else.
- 2. Plots of contaminated signal, ideal signal, DFT of contaminated, and filtered signal after you have zeroed the appropriate samples in the DFT, for the ECG signal of length Ns = 512.
- 3. Same set of plots for the last ECG signal. "Last ECG Signal" should appear on the display when you are working on the last ECG signal.
- A report containing a tabulation of the results of your calculations for the indices of the samples zeroed. Show the work for the calculation when Ns = 512, and explain why the noise cannot be completely removed.
- 5. Explain why other signals could be successfully filtered.