Biomedical Instrumentation Spring, 2003

Solutions to Problem Set 2

1. The mercury-and-glass thermometer measures temperature by determining the increase in volume of a small volume of mercury by having the mercury expand into a fine-bore glass capillary tube. The expanded volume of the mercury forms a cylinder of mercury in this tube. The height of the cylinder is proportional to be increase in volume which in turn is proportional to the temperature. Thus, we can consider this thermometer as a temperature-to-length transducer. The sensitivity of the transducer can be determined from the scale given in the drawing. First, we must measure of the length of the calibrated scale. The distance between the 20 degree and the 30 degree marks is 75 mm. Therefore, the sensitivity will be 7.5 mm/degrees Celsius.

The range of this instrument can be determined from the scale shown on the drawing. The mercury column can be measured between 20 and 30 degrees Celsius, so that is the range of the instrument.

The precision of this instrument can be estimated by looking at the scale. Since the scale is marked every half degree we know that we can see changes in temperature of a half degree reliably. Since there are almost four mm between individual marks on the scale, we probably could estimate when the mercury column is halfway between two marks without a great deal of difficulty. Thus, we can estimate the precision of this instrument to be 0.25 degrees C.

The resolution of the thermometer is concerned with how small a change in temperature we can see with this device. Of course we can see a change of half a degree since the mercury column will change its length by one scale marking. Actually, we can do much better since we should be able to see a change in length of the mercury column of roughly one mm, or perhaps even less. Thus, our resolution could be approximately

$$\text{Resolution} = \frac{1\text{ mm}}{7.5\text{ mm}} \text{ x } 1°\text{C} = 0.13°\text{C}$$

2. The power spectral analysis of the signal found its highest frequency to be 85 Hz. This means that to fully reproduce this signal we need to sample at least at twice this frequency or 170 Hz. In practice if we sampled at this frequency we still would not be able to fully reproduce the signal, and so we should follow the rule of thumb that states that we should sample at approximately five times the highest frequency component of the signal. This would give a sampling rate of 425 Hz.

Once we sampled at the appropriate to frequency and we then reconstructed the signal, we would find that a power analysis would provide us with the original information plus some additional high frequency information that was not in the original signal. This high frequency component would be the result of reconstructing the signal from the sampled points and the artifactually introduced signal between the sampling points. If the reconstruction algorithm fits the points to a smoothing curve, this artifact will be diminished, but it is usually still present. Since this artifact is at a frequency that is higher than the highest frequency component of the signal, it can often be removed by passing of the reconstructed signal through a low pass filter with a cutoff frequency of 85 Hz or a little higher, but not as high as 170 Hz.

3. If the practical spacing of turns of resistance wire on a resistance element of a linear or angular variable resistance displacement transducer is 20-40 turns per mm, then the resistance value of this variable resistor can be divided into 20-40 steps per mm. For the linear displacement transducer this becomes 1 mm divided by 20 or 40 to get the resolution.

Best Resolution = 1 mm/40 = 0.025 mm

When there are only 20 turns per mm

Resolution = 1 mm/20 = 0.05 mm

When we are measuring angular displacement, we must form the resistance element into a circle. We can now relate 1 mm displacement along this element to angle by

Angle in radians = $(1 \text{ mm} \times 2\pi)/\pi D$ Angle in degrees = $(1 \text{ mm} \times 360)/\pi D$

Where D is the diameter of the circular resistance element.

Using the values for the linear resolution, the angular resolution becomes

Angular resolution = $(1 \text{ mm} \times 360)/40\pi D = 2.86/D$ degrees

Or

Angular resolution = $(1 \text{ mm} \times 360)/20\pi D = 5.73/D$ degrees