Biomedical Instrumentation Spring, 2003

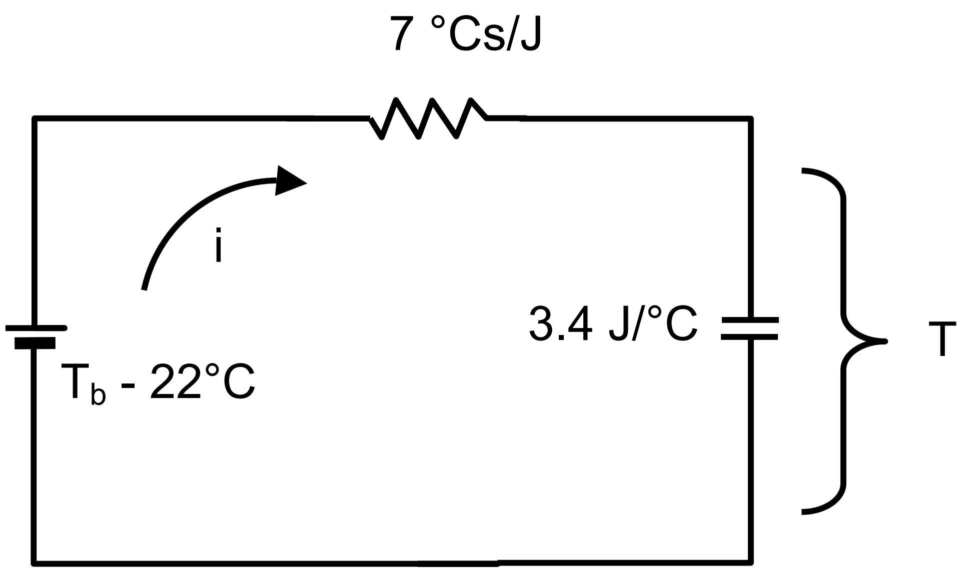
Solutions to Problem Set 5

1. The biologic tissue can be considered as an infinite heat sink in that it has a much larger mass than the temperature sensor. Thus we can consider it to be a heat source at a constant temperature of T_b. In electrical terms, this is equivalent to a constant voltage source. In the circuit below, the actual voltage source will be the difference between T_b and the initial temperature of the temperature sensor. The temperature sensor has a thermal capacity of

$C = mC_s = 2g \bullet 1.7 \, {}^{J}\!/_{g°C} = 3.4 \, {}^{J}\!/_{°C}$ In electrical terms, this is equivalent to a capacitor

The thermal resistance between the heat source and the temperature sensor is given as 7 °Cs/J. This is equivalent to an electrical resistor.

Thus, our circuit becomes:



Thus, we can find the temperature of the sensor as a function of time, T(t) by recognizing that this is a simple first-order RC circuit that has a time constant of τ = RC. The equation is then

$$T(t) = (T_b - 22°C)e^{\frac{-t}{7°Cs/J)(3.4J/°C}} = (T_b - 22°C)e^{\frac{-t}{23.8s}}$$

It takes 5 time constants to reach 99% of the final value. Since we do not know T_b, we cannot find exactly what the time to reach within 0.5 °C, but since biologic tissue is usually at a temperature of about 37 °C, we should be within 0.5 °C in 5 time constants. Thus the time should be 5 x 23.8 s = 119 s.

2. For this circuit, full-scale current is 50 µA. When $R_x = \infty$ the current through the meter should be 50 µA. The total resistance in the circuit is R_s + 1,000 Ω, so we can solve for R_s by

$$R_s + 1{,}000\Omega = \frac{3V}{50\mu A} \quad \Rightarrow \quad R_s = 60{,}000\Omega - 1{,}000\Omega = 59{,}000\Omega$$

For the meter to read half scale (25 μA) the voltage across the meter and its 1,000 Ω resistance must be

$$V_m = 25\mu A \bullet 1{,}000\Omega = 25mV$$

So the voltage drop across R_s has to be 3V – 25 mV = 2.975 V, so the current through this resistor will be

$$I = \frac{E}{R} = \frac{2.975V}{59{,}000\Omega} = 50.4\mu A$$

Since 25 μA of this current goes through the meter, the remainder goes through R_x. This current is 50.4 μA – 25 μA = 25.4 μA.

Thus since we know the voltage across R_x and the current through it, we can determine its value

$$R_x = \frac{25mV}{25.4\mu A} = 983\Omega$$

Note that this resistance is almost equal to the meter resistance. This makes sense since approximately half of the current from the battery should go through the meter and approximately half should go through the unknown resistance being determined.