1. We are using a thermistor and a PN junction diode in the opamp circuit shown below. The resistance of the thermistor as well as the current-voltage relationship of the PN junction diode are given as follows

\[ R(T) = R_0 e^{\beta \left( \frac{1}{T} - \frac{1}{T_0} \right)} \quad \text{and} \quad I = A e^{\frac{q(V-E_s)}{kT}} \]

Find the expression for the output voltage of the circuit in terms of the temperature of the thermistor. Please note that the PN junction diode remains at fixed temperature of T_0 at all times.
We know that the current on the thermistor is equal to the current on the P-N junction diode

\[ I = \frac{1}{P_0} e^{-\frac{B}{T} \left( \frac{1}{T} - \frac{1}{T_0} \right)} = A e^{\frac{qV}{kT_0} - \frac{E_g}{kT_0}} \]

\[ \ln(A R_0) = -B \left( \frac{1}{T} - \frac{1}{T_0} \right) - \frac{qV}{kT_0} + \frac{E_g}{kT_0} \]

\[ = C_0 = \frac{R_0}{T} - \frac{qV}{kT_0} \quad \text{where} \quad C_0, C_1, \text{and} \ C_2 \text{are constants.} \]

\[ = V = \frac{C_1}{T} + C_2 \]

(See problem #2)
2. Assume that the output voltage of a sensor is given as below

\[ V_1(T) = \frac{c_1}{T} + c_2, \] which is a non-linear relationship.

In the narrow region of interest, \( T_1 - T_2 \), we would like to use the relationship \( V_2(t) = K_1 T + K_2 \), to approximate the non-linear relationship above. Show the steps to be followed to find the maximum error from this approximation.
At the limits of the range, two functions must have the same values, i.e.

\[ V_1(T_1) = V_2(T_1) \implies \frac{C_1}{T_1} + C_2 = k_1 T_1 + k_2 \]
\[ V_1(T_2) = V_2(T_2) \implies \frac{C_1}{T_2} + C_2 = k_1 T_2 + k_2 \]

Solve for \( k_1 \) and \( k_2 \)

Note that the slope of \( V_2(T) \) is negative, i.e. \( k_2 \) must be negative.

Error \( E(T) = V_2(T) - V_1(T) = \frac{C_1}{T} + C_2 - k_1 T - k_2 \)

Error is maximum when

\[ \frac{dE(T)}{dT} = 0 \implies - \frac{C_1}{T^2} - k_1 = 0 \implies T_0 = \sqrt{\frac{C_1}{k_1}} \]

is the temp where the error is max.

Max Error \( E(T_0) = \sqrt{-k_1 C_1 + C_2} - \sqrt{-C_1 k_1 - k_2} = C_2 - k_2 \)
3. Show that the gain of a non-inverting amplifier is given as

\[ \frac{Gain}{V_{in}} = \frac{V_{out}}{V_{in}} = 1 + \frac{Z_{out}}{Z_{in}} \]

\[ V_+ = V_{in} \]
\[ V_- = V_{out} + \frac{Z_{in}}{Z_{in} + Z_{out}} \]
\[ V_{out} = A \left( V_+ - V_- \right) = A V_{in} - A V_{out} \frac{Z_{in}}{Z_{in} + Z_{out}} \]
\[ V_{out} \left( 1 + A \frac{Z_{in}}{Z_{in} + Z_{out}} \right) = A V_{in} \]
\[ \frac{V_{out}}{V_{in}} = \lim_{{A \to \infty}} A \frac{Z_{in}}{Z_{in} + Z_{out}} = 1 + \frac{Z_{out}}{Z_{in}} \]
4. What does the following amplifier do?

\[ V_{out} = -R_3 \left( I_1 + I_2 \right) \]

This circuit will come in handy in problem #6
5. We are mapping the temperature of a patient using IR radiometry. Our scanning area is 1 meter wide and half a meter high. Our sensor is very slow, and responds after 1 second of exposure. If we want to complete the scan of the entire area in 1 minute, what is the largest pixel size we can have?

\[ \text{Area} = 0.5 \text{ m}^2 \]

\[ \text{pixel area} = \frac{0.5}{60} = 8.3 \text{ cm}^2 \approx 13 \text{ inch}^2 \]

Not very good resolution would be nice to increase it by speeding up the sensor (see problem 6)
6. Assume that the step response of the sensor above was given as

\[ g(t) = 1 - e^{-t} \]

Design a system to speed this sensor up, i.e. try to get a step pulse out of the sensor when a step pulse is fed to its input. Show the mathematical derivation followed by the physical implementation.
7. We would an amplifier to pass signals from DC (0 Hz) to 100 Hz, and reject signals above 100 Hz. Design a 1st order active Low Pass Filter, with DC gain of 10 and input impedance of 10 KΩ.
\[
\text{Gain} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\frac{1}{j\omega C R_2}}{R_1 + j\omega C R_2}
\]

\[
= \frac{\frac{1}{j\omega C R_2}}{R_1 + j\omega C R_2}
\]

\[
R_1 = 10 \, \text{K}\Omega
\]

At DC, \( f = 0 \) Hz, \( \text{Gain} = \frac{R_2}{R_1} = 10 \)

\( \Rightarrow R_2 = 100 \, \text{K}\Omega \)

Corner Freq: \( R_1 = \omega C R_1 \, R_2 \)

\[= \] \( 1 = \omega C R_2 \)

\[= \] \( \omega_c = \frac{1}{CR_2} \)

\[= \] \( f_c = \frac{1}{2\pi CR_2} = 15.9 \, \text{Hz} \)